Chapter 10

TIME SERIES ANALYSIS: SMOOTHING TECHNIQUES

Time Series

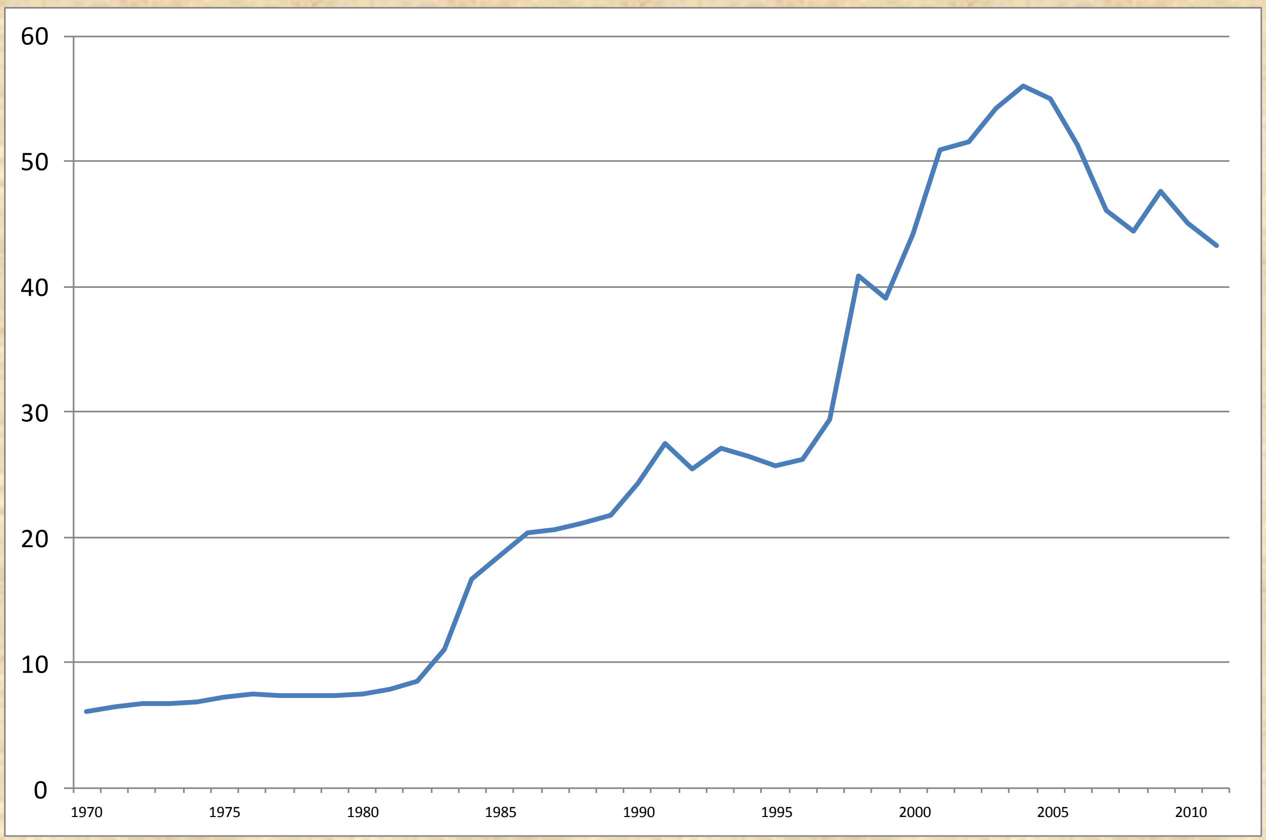
Definition:

A ***time series*** is a sequence of n observations $Y_1, Y_2, \ldots, Y_n$, on a process at equally spaced points in time (e.g., daily, monthly, annual, etc).

Example: Exchange Rate (PhP/US$), 1970-2009

Year	Php/US$	Year	Php/US$
1970	6.0246	1990	24.3105
1971	6.4317	1991	27.4786
1972	6.6749	1992	25.5125
1973	6.7563	1993	27.1199
1974	6.7887	1994	26.4172
1975	7.2479	1995	25.7144
1976	7.4403	1996	26.2157
1977	7.4028	1997	29.4707
1978	7.3658	1998	40.8931
1979	7.3776	1999	39.089
1980	7.5114	2000	44.1938
1981	7.8996	2001	50.9927
1982	8.54	2002	51.6036
1983	11.1127	2003	54.2033
1984	16.6987	2004	56.0399
1985	18.6074	2005	55.0855
1986	20.3857	2006	51.3143
1987	20.5677	2007	46.1484
1988	21.0948	2008	44.4746
1989	21.7367	2009	47.6372

Graph of Exchange Rate: 1970-2009



Objectives of Time Series Analysis

- To fit a model and proceed to forecasting and monitoring
- To identify the individual components of a time series

Components of Time Series

- TREND: describes the long-term movement in a time series and usually modeled by a smooth curve. It is the underlying direction (an upward or downward tendency) that is the result of influences such as population growth, price inflation and general economic changes.

- SEASONAL: describes the short-term recurring pattern of change in the series and consists of relatively repetitious cycles of fixed amplitude and duration. Seasonality occurs when the time series exhibits regular fluctuations during the same month (or months) every year, or during the same quarter every year. For instance, retail sales peak during the month of December. It could be attributed to (i) natural conditions such as weather fluctuations that are representative of the season, (ii) business and administrative procedures such as the start and end of the school term; or, (iii) social and cultural behavior such as christmas.

- CYCLICAL: movements in a time series that, like seasonal variations, are recurrent but that, unlike seasonal variations, occur in cycles longer than one year. This pattern exists when the series is influenced by longer-time economic fluctuations such as those associated with the business cycle (usually around 5 years). The major distinction between the seasonal and cyclical components is that the seasonal component is of a constant length and recurs on a regular basis, while the cyclical component varies in length and magnitude.

- IRREGULAR: describes the miscellaneous erratic movements in the series and tends to have an irregular, saw-toothed pattern. This component cannot be predicted with certainty. The irregular component is present in every time series that makes it a random variable.

Smoothing Techniques

- ***Smoothing techniques*** are used to reduce the short-term fluctuations in the time series (irregular and seasonal components). This technique, when properly applied, reveals more clearly the underlying long-term movements in the series (leaving out the trend and cyclical components).

- General Methods:
  - Averaging Methods: past observations are given equal weights in evaluating the forecast (ex: single moving average)
  - Exponential Smoothing Methods: past observations are given unequal weights that decay exponentially (ex: single exponential smoothing)

Single Moving Average

Let Y_t be the variable at time t and S_t be the smoothed measure at time t.

Step 1. Choose the number of periods (T) to be used in the computation of the forecast.
Step 2. Compute for the moving average (smoothed measure) using the following formula:

$$S_T = \frac{\sum_{t=1}^{T} Y_t}{T}, S_{T+1} = \frac{\sum_{t=2}^{T+1} Y_t}{T}, \ldots, S_{T+k} = \frac{\sum_{t=k+1}^{T+k} Y_t}{T}$$

Notes:

- The moving average cannot be computed for the first (T – 1) periods.
- The oldest observation is dropped as each new observation becomes available.
- The larger the value of T, the greater the smoothing effect and less sensitive to changes in the value of Y. The smaller the value of T, the more the moving averages will follow the pattern of the data set and more sensitive to changes in the value of Y.

Example

T=5

$$S_5 = \frac{\sum_{t=1}^{5} Y_t}{5} = \frac{6.0246 + 6.4317 + 6.6749 + 6.7563 + 6.7887}{5} = 6.5352$$

$$S_6 = \frac{\sum_{t=2}^{6} Y_t}{5} = \frac{6.4317 + 6.6749 + 6.7563 + 6.7887 + 7.2479}{5} = 6.7799$$

T=10

$$S_{10} = \frac{\sum_{t=1}^{10} Y_t}{10} = \frac{6.0246 + 6.4317 + 6.6749 + 6.7563 + 6.7887 + 7.2479 + 7.4403 + 7.4028 + 7.3658 + 7.3776}{10} = 6.95106$$

$$S_{11} = \frac{\sum_{t=2}^{11} Y_t}{10} = \frac{6.4317 + 6.6749 + 6.7563 + 6.7887 + 7.2479 + 7.4403 + 7.4028 + 7.3658 + 7.3776 + 7.5114}{10} = 7.09974$$

Example: Exchange Rate (PhP/US$), 1970-2009

Period	Year	Php/US$	Period	Year	Php/US$
1	1970	6.0246	21	1990	24.3105
2	1971	6.4317	22	1991	27.4786
3	1972	6.6749	23	1992	25.5125
4	1973	6.7563	24	1993	27.1199
5	1974	6.7887	25	1994	26.4172
6	1975	7.2479	26	1995	25.7144
7	1976	7.4403	27	1996	26.2157
8	1977	7.4028	28	1997	29.4707
9	1978	7.3658	29	1998	40.8931
10	1979	7.3776	30	1999	39.089
11	1980	7.5114	31	2000	44.1938
12	1981	7.8996	32	2001	50.9927
13	1982	8.54	33	2002	51.6036
14	1983	11.1127	34	2003	54.2033
15	1984	16.6987	35	2004	56.0399
16	1985	18.6074	36	2005	55.0855
17	1986	20.3857	37	2006	51.3143
18	1987	20.5677	38	2007	46.1484
19	1988	21.0948	39	2008	44.4746
20	1989	21.7367	40	2009	47.6372

Using Data Analysis ToolPak

Step 1: Enter time series data in one column.

Step 2: Choose Data→Data Analysis →Moving Average

Step 3: Fill up dialogue box.

Moving averages are used to emphasize the direction of a trend and to smooth out fluctuations in the values of Y, or "noise", that can confuse interpretation. Typically, upward momentum is confirmed when a short-term average crosses above a longer-term average. Downward momentum is confirmed when a short-term average crosses below a long-term average. For example, in monitoring the movement of stock prices, investors maintain a short-term moving average (T=15 days) and a longer-term moving average (T=50). When the two cross each other then this indicates a change in the direction of the trend.

Single Exponential Smoothing

Step 1. Choose the weight α (between 0 and 1).

Step 2. Decide on the initial value (S_2) of the smoothed series. Possible choices: Y_1, mean of the values in the series, or mean of the first T values in the series.

Step 3. Compute for the smoothed value using the formula:

$$S_{t+1} = \alpha Y_t + (1 - \alpha) S_t$$

Notes:

- An α value close to 1 has very little smoothing effect, whereas an α value close to 0 has considerable smoothing effect.
- The smaller the value of α, the more sensitive the resulting smoothed series to the choice of initial value.

Remarks

The smoothed value can be expressed as follows:

$$S_{t+1} = \alpha Y_t + (1-\alpha)(\alpha Y_{t-1} + (1-\alpha)S_{t-1})$$
$$= \alpha Y_t + (1-\alpha)\alpha Y_{t-1} + (1-\alpha)^2\alpha Y_{t-2} + (1-\alpha)^3\alpha Y_{t-3} + \ldots$$

The weights of the past observations decrease exponentially.

Dampening is fast when α is large.

Observation	Y_t	Y_{t-1}	Y_{t-2}	Y_{t-3}
Weights	α	$\alpha(1-\alpha)$	$\alpha(1-\alpha)^2$	$\alpha(1-\alpha)^3$
	0.9	0.09	0.009	0.0009
	0.6	0.24	0.096	0.0384
	0.4	0.24	0.144	0.0864
	0.1	0.09	0.081	0.0729

Example

α=0.8

$$S_2 = Y_1 = 6.0246$$

$$S_3 = (0.8 * 6.4317) + (0.2 * 6.0246) = 6.35028$$

$$S_4 = (0.8 * 6.6749) + (0.2 * 6.35028) = 6.609976$$

α=0.2

$$S_2 = Y_1 = 6.0246$$

$$S_3 = (0.2 * 6.4317) + (0.8 * 6.0246) = 6.10602$$

$$S_4 = (0.2 * 6.6749) + (0.8 * 6.10602) = 6.219796$$

Example: Exchange Rate (PhP/US$), 1970-2009

Period	Year	Php/US$
1	1970	6.0246
2	1971	6.4317
3	1972	6.6749
4	1973	6.7563
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Period	Year	Php/US$
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35	2004	56.0399
36	2005	55.0855
37	2006	51.3143
38	2007	46.1484
39	2008	44.4746
40	2009	47.6372

Using Data Analysis ToolPak

Step 1: Enter time series data in one column.

Step 2: Choose Data→Data Analysis →Exponential Smoothing

Step 3: Fill up dialogue box. Damping factor = 1-α

- The smoothing techniques can also be used for forecasting purposes. This approach studies the deterministic (trend, cycle, seasonality) components only. No minimum sample size is required for these methods
- Stat 145 will focus on Box-Jenkins models . These methods will concentrate on the irregular component of the time series. These methods require at least 36 observations for seasonal data and at least 50 for non-seasonal data.