Statistics 115
Sample First Exam   TGCapistrano

I.   TRUE OR FALSE. Write 'TRUE' if the statement is always true; otherwise, write 'FALSE'.
1.   If A, B, and C are events then $P(A \cup B \cup C) = P(A) + P(B) + P(C)$. (FALSE)
2.   If we toss a coin twice and A=event of observing 2 heads, then $P(A)=1/4$. (FALSE)
3.   If A is the impossible event then the probability of event A is always equal to 0. (TRUE)
4.   If X is a discrete random variable and 0 is not a mass point then $P(X=0)=0$. (TRUE)
5.   If X is a continuous random variable then $P(X=0)$ is always equal to 0. (TRUE)
6.   When a sample of size n is selected using simple random sampling with replacement (SRSWR) and X_i is the measure taken from the ith selected element in the sample then (X_1, X_2, …, X_n) is a random sample from an infinite population. (TRUE)
7.   When a sample of size n is selected using simple random sampling without replacement (SRSWOR) and X_i is the measure taken from the ith selected element in the sample then (X_1, X_2, …, X_n) is a random sample from an infinite population. (FALSE)
8.   The sample mean of a random sample of size n taken from a normal distribution will also follow a Normal distribution even if the sample size is small. (TRUE)
9.   The Central Limit Theorem requires that the parent population from which the random sample comes from is normally distributed for the distribution of the sample mean to approximate a normal distribution when the sample size is sufficient large. (FALSE)
10.   The t-distribution approaches the standard normal distribution as its degrees of freedom approaches infinity. (TRUE)

II.   FILL IN THE BLANKS.
1.   If we select a sample of size 5 from a population containing 50 elements using SRSWR, then the number of ordered 5-tuples in the sample space is _____.
2.   If we select a sample of size 5 from a population containing 50 elements using systematic sampling, then the number of ordered 5-tuples in the sample space is _____.
3.   If we select a sample of size 25 from a population containing 125 elements using SRSWR from a population whose $\mu=400$ and $\sigma^2=100$, then the value of the standard error of the sample mean is _____.
4.   If we select a sample of size 25 from a population containing 125 elements using SRSWOR from a population whose $\mu=400$ and $\sigma^2=100$, then the value of the standard error of the sample mean is _____.
5.   The formula to compute the empirical $P(A)$ using the a posteriori approach is _____.
6.   If X is a random variable and $F(\cdot)$ is its cumulative distribution function (CDF) then for any real number x, $F(x) =$ _____.
7.   If X is a discrete random variable and $f(\cdot)$ is its probability mass function (PMF) then for any real number x, $f(x) =$ _____.
8.   If Z is a random variable that follows a standard normal distribution then the mean of Z is equal to a) _____ and the variance of Z is equal to b) _____.
9.   If Z is a random variable that follows a standard normal distribution and α is a real number between 0 and 1 then $P(-z_{\alpha/2} < Z < z_{\alpha/2}) =$ _____.
10.   If X is a random variable that follows an F-distribution with numerator and denominator degrees of freedom equal to 10 and 15, respectively, then $F_{0.99}(v_1=10, v_2=15)=$_____.

III. PROBLEM SOLVING. Show all the important steps. Always show the formulas used in answering the following problems with the appropriate values plugged-in.

1. There are 8 elements in the population and each one is assigned a serial number from 1 to 8. The elements labeled 1, 2, 3 are all males, while the elements labeled 4, 5, 6, 7, 8 are all females. Suppose a sample of size 4 is selected from this population using SRSWOR.
   a) Specify the sample space by roster method using the notation, $\{\omega_1, \omega_2, \omega_3, \omega_4\}$ where the ω_is are the serial numbers of the selected elements, to denote a sample of size 4.
   b) Let A= event of selecting 2 males in the sample and B=event of selecting less than 2 females in the sample. Specify events A and B by roster method.
   c) Compute for P(A) and P(B).

2. Suppose X is a continuous random variable and its CDF is:

$$F(x) = \begin{cases} 0 & \text{when } x \leq 0 \\ \dfrac{x^2}{2} & \text{when } 0 < x \leq 1 \\ \dfrac{4x - x^2 - 2}{2} & \text{when } 1 < x \leq 2 \\ 1 & \text{when } x > 2 \end{cases}$$

   Compute for the following probabilities:
   a) P(X < 1.25)   c) P(-5 < X < 1.1)
   b) P(X > 0.4)   d) P(0.2 < X ≤ 2)

3. Suppose X is a discrete random variable and its PMF is:

x	-2	2	4	6
f(x)	1/10	3/10	4/10	2/10

   Compute for the mean and variance of X.

4. The temperature X (in °C) at noon on any day in March in Quezon City is known to follow a normal distribution with mean 36.5°C and variance of 4.0°C. Evaluate the probabilities of the following events for the weather forecaster who wishes to report them for his forecast in television:
   a) event that the temperature is more than 38.4 oC
   b) event that the temperature is between 35.5 oC and 38.5 oC

5. A random sample of size 400 is selected from a large population whose mean and variance are μ=250 and σ^2=144, respectively. Approximate $|P\ |\bar{X} - \mu| \leq 0.672$.

6. A soft-drink machine is regulated so that the amount of drink dispensed is normally distributed mean equal to 200 ml per bottle and standard deviation equal to 10 ml. Periodically, the machine is checked by taking a random sample of 16 bottles and computing for the sample mean.
   a) Assuming that the machine is in perfect condition so that it meets all specifications, what is the probability of selecting a sample whose sample mean is greater than 225.5 ml?

b) Assuming that the machine is in perfect condition so that it meets all specifications, what is the probability of selecting a sample whose sample variance is greater than 166.64 ml2?

c) Assuming that something is wrong with the machine so that the population mean is actually 186.99 ml. and the population variance is unknown, what is the probability of selecting a sample whose sample mean is less than 200 if the standard deviation of the selected sample is equal to 20?