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Abstract

Philippine seasonal time series data tends to have unstable seasonal behavior, called seasonal volatility. Current Philippine seasonal adjustment methods use X-11-ARIMA, which has been shown to be poor in the presence of seasonal volatility. A modification of the Census X-11 method for seasonal adjustment is devised by changing the moving average filters into median-based filtering procedures using Tukey repeated median smoothing techniques. To study the ability of the new procedure, simulation experiments and application to real Philippine time series data were conducted and compared to Census X-11-ARIMA methods. The seasonal adjustment results will be evaluated based on their revision history, smoothness and accuracy in estimating the non-seasonal component.The results of research open the idea of using robust nonlinear filtering methods as an alternative in seasonal adjustment when moving average filters tend to fail underunfavorable conditions of time series data.

Keywords: Tukey Median Smoothing, Unstable Seasonality, Seasonal Filtering, Census X-11-ARIMA, Robust Filtering

AMS Classification: **62G35, 91B84, 60G35,62M10, 62M20, 62P20**

JEL Classification: C82, C14, C22, C49

Introduction

Seasonal adjustment of time series data involves removing regularly observed seasonal patterns to unmask hidden trends, cycles, and other characteristics hidden by seasonal factors (Ghysels and Osborn, 2001). Removing these features opens the time series data to further investigation and analysis by end-users with minimum loss of information on the time series (Bell and Hillmer, 2002). Statistical agencies publish seasonally-adjusted data for public use and for evaluating leading economic indicators to forecast future directions of economic growth (National Statistical Coordination Board, 2011). In addition, seasonal adjustment of economic time series variables before econometric modeling removes the seasonality as an intervening variable, thus avoiding conclusions from spurious relationships (Granger, 1979).

The problem of seasonal adjustment programs is in the estimation of seasonal factors when nonlinearities in time series patterns exist, especially on the seasonal behavior. The non-constant pattern of seasonal behavior in time series is called "seasonal volatility" (Bersales, 2010) or "moving seasonality" (Higginson, 1975), where seasonal patterns tend to be different in magnitude among years. Researches on seasonal adjustment of Philippine time series datahave concluded that seasonal volatility are present (Redoblado, 2005) and these seasonal volatilities result to poor seasonal adjustment results for X11-ARIMA88 (Bersales, 2010).

Robust filtering methods are used to extract patterns when nonlinear behavior exists in time series data. Tukey (1977) introduced median smoothing methods to extract smooth patterns hidden by nonlinear rough spikes in time series data. Through median filtering, any volatile behavior would be smoothed out from trend and seasonal behaviors.

From the established background of research, the paper devises a modification of the X-11 seasonal adjustment methodology by replacing the moving average filters with Tukey median smoothing to make seasonal adjustment results more robust in the presence of seasonal volatility. The devised procedure is compared with default X-11 seasonal adjustment using evaluation measures based on principles of (1) minimum revision of seasonally-adjusted data, (2) smoothness, and (3) accuracy in estimation. The procedure is applied to Philippine seasonal time series data for demonstration and comparison with default X-11 results.

Concept of Seasonality and Seasonal Adjustment

Seasonal time series are decomposed into two major components: (1) the non-seasonal components and (2) seasonal components (Pierce, 1980; Ghysels and Osborn, 2001; Bell and Hillmer, 2002). Non-seasonal components are the trend, cycle, and irregular behavior of the time series, with the first two commonly combined as trend-cycle component (Pierce, 1980; Ghysels and Osborn, 2001; Hyndman, et al., 2008; Singapore Department of Statistics, 2006). Seasonal components are the crude sub-annual regular pattern of the data, or calendar day effects such as holidays and trading days (Ghysels and Osborn, 2001; Singapore Department of Statistics, 2006). These components are unobserved because the individual behavior of each component is unknown, yet their total overall behavior results to the observed time series data (Bell and Hillmer, 2002).

Trend-cycle is the long-term pattern of increase or decrease in magnitude of the time series data (Singapore Department of Statistics, 2006). Cycles are the long-term wave patterns of time series data with peaks and troughs spanning at least one year and trend is the long-run direction of growth or decay of the time series (Hyndman, et al., 2008).

Irregularities are the residuals of the time series data after the trend and seasonal patterns have been accounted and extracted (Branch and Mason, 2006). No pattern can be ascertained from this component and is generally mean-stationary (Singapore Department of Statistics, 2006).

Seasonal patterns are regular patterns observed within every year (Branch and Mason, 2006). These can be crude seasonal patterns such as visitor arrivals in a certain destination that peak on certain months of the year (Singapore Department of Statistics, 2006), or due to calendar days of the year, such as holidays and working days which affect production statistics within a year (Branch and Mason, 2006; Singapore Department of Statistics, 2006).

The observed time series data is generated by the interaction of the unobserved time series component through the following structures: (1) additive, (2) multiplicative, and (3) log-additive. Letting y_t be the original series observed at time *t* which is decomposed into the trend-cycle TC_t , seasonal factor S_t, and irregular term I_t, the form for the following structures are shown below (Pierce, 1980; Ghysels and Osborn, 2001):

Additive Decomposition: $$y_t = TC_t + S_t + I_t \tag{1}$$

Multiplicative Decomposition: $$y_t = TC_t \times S_t \times I_t \tag{2}$$

Log-Additive Decomposition: $$\ln y_t = \ln TC_t + \ln S_t + \ln I_t \tag{3}$$

To produce the seasonally-adjusted time series data, the seasonal term is estimated and removed by the appropriate mathematical operation; e.g., subtract the seasonal term for additive structure or division for multiplicative structures.

Seasonal Volatility

Philippine economic time series data were noted to have unstable seasonal behavior (Redoblado, 2005). These unstable seasonal behavior are called seasonal volatility (Bersales, 2010) or as moving seasonality (Higginson, 1975) where seasonal pattern tend to have (1) different magnitudes yet same direction or (2) an slow transition of the seasonal changes to another form. It is in seasonal volatility that affects the estimation of seasonal factors that leads to poor seasonal adjustment results (Bersales, 2010).

Seasonal volatility is simulated using the seasonal generalized autoregressive conditional heteroscedasticity model (SGARCH) in Ghysels and Osborn (2001). For every $i = 1, 2, ..., S$, where S is the number of seasons in a year, the $SGARCH(p, q, S)$ model for the conditional variance $h_{i,t}$ at year t in season i is shown below:

$$\text{SGARCH:} \quad \begin{aligned} &\varepsilon_{i,t} = v_{i,t}\sqrt{h_{i,t}} \\ &v_{i,t} \sim White\ Noise(0,1) \\ &h_{i,t} = \alpha_{i,0} + \sum_{j=1}^{p}\alpha_{i,j}\varepsilon_{i,t-jS}^{2} + \sum_{k=1}^{q}\beta_{i,k}h_{i,t-kS} \end{aligned} \tag{4}$$

For $S = 1$, the SGARCH model becomes the $GARCH(p,q)$ model of Bollerslev (1986). On the third equation, the first summation term deals with the short-term seasonal effects or 'shocks' to conditional variance, whilst the second summation deals with the pattern of long-term seasonal variation (Tsay, 2002; Ghysels and Osborn, 2001).

In the presence of seasonal volatility for time series data, X-11 programs are poor in terms of estimation for seasonal adjustment (Ghysels, et al., 1997). The modification of X-11 through median smoothing is sought as a robust means for estimation and improvement of seasonal adjustment in the presence of volatile seasonal behavior.

Median Smoothing Methods

Median filtering was introduced by Tukey (1977) as a procedure to smooth data gathered from equally-spaced linearly-ordered intervals, such as for every year, for every quarter, for every month, or every mile of road, every feet of height, and so on. The methods of median smoothing use notations listed in the table below.

Tukey (1977) did not make any theoretical study for median smoothing, but Gallagher and Wise (1981) have concluded in their theoretical analysis that iterated median filtering preserve level shifts and constant signals. Thus, after the application of repeated median filters, what is left to the smooth is the common trend signal and the level shifts. If there is locally increasing or decreasing trend, hanning and skip mean procedures are used as steps in smoothing. In terms of local polynomial trends, median filters are able to approximate such patterns, but flats out for polynomial curves shorter than the window of the smoother (Rabiner, et al, 1975).

In the distributional assumptions of the median filter, it is known to be optimal in the Laplace family of distributions (Arce, 2005). Weighted median filters, to which median filters are special cases, are optimal in the case of generalized Gaussian distributions, also known as the generalized error distribution, or the generalized Laplace distribution (Arce, 2005), which have the normal and Laplace distribution as special cases, and which the distribution family is described as heavy-tailed, making extreme distribution values more frequent and the distribution of values more volatile than those in the normal case. Assuming Laplace optimality makes median filters relatively resistant to impulsive noise.

In minimum risk optimality, median filters are optimal in terms of linear loss functions, analogous to where moving averages are optimal to quadratic error losses. The median filter is the optimal minimum risk estimator for data $\left(x_{t-k},...,x_t,...,x_{t+k}\right)$ with the loss function of the form (Bouman, 2010):

$$\text{Linear Loss Function:} \quad L\left(\theta \mid x_{t-k},...,x_t,...,x_{t+k}\right) = \sum_{r=-k}^{k}\left|x_{t+r} - \theta\right| \tag{5}$$

The software used to generate median filtering results is the Excel add-in of Huber (2004). The standard notation of Tukey (1977) is used by the software as a standard in which filter to be used.

Table 1. Symbols of Tukey Median Smoothing

Symbol	Meaning	Example
3, 5, 7, 9, 11,… , i.e., any odd number	Window of the centered median smooth, e.g. k-term median smooth: $Y_t^{smooth} = median\left\{X_{t-\frac{k-1}{2}}, X_{t+1-\frac{k-1}{2}}, ..., X_t, ..., X_{t-1+\frac{k-1}{2}}, X_{t+\frac{k-1}{2}}\right\}$ Where X= input series, Y=smoothed series	"3" = 3-term median smooth $Y_t^{smooth} = median\{X_{t-1}, X_t, X_{t+1}\}$ "5", "7",…
R	Repeat smoothing with same width until convergence of smooth results.	"3R", "5R", "9R"
' (apostrophe)	Copy end values in 3-term median smoothing. In median smoothers of width greater than 3, copying is default. In 3-term smoothing, the default smoothing is shown below: In the beginning: $Y_1^{smooth} = median\{3Y_2^{smooth} - 2Y_3^{smooth}, X_1, Y_2^{smooth}\}$ In the end: $Y_T^{smooth} = median\{Y_{T-1}^{smooth}, X_T, 3Y_{T-1}^{smooth} - 2Y_{T-2}^{smooth}\}$	"3R' "
S	Splitting the smoothed series into subseries from points of flat peaks and flat troughs created by data points with similar smoothed values. For each subseries, a "3R" smooth is applied. Tukey (1977) suggests splitting to be done twice ("SS")	"3RSS", "5RSS",
>	Skipping mean filter. A moving average filter with weights (0.5,0,0.5): $Y_t^{Skip} = \frac{1}{2}Y_{t-1}^{smooth} + \frac{1}{2}Y_{t+1}^{smooth}$	"3RSS>", "5R>","7>"
H	Hanning filter. A moving average filter with weights (0.25,0.5,0.25): $Y_t^{Hann} = \frac{1}{4}Y_{t-1}^{smooth} + \frac{1}{2}Y_t^{smooth} + \frac{1}{4}Y_{t+1}^{smooth}$	"3RSSH", "5RH", "7H"
, (comma)	Indicates separate smoothing for "re-roughing" procedures. Re-roughing means smoothing the rough residuals of an initial smoothing procedure using another step of smoothing. The smoothed values of the residuals are added to the initially derived smoothed data as the final smooth. Twicing is repeating the same initial smoothing procedure to smoothing rough residuals. Other repetitive procedures may be thricing, 4-cing, 5-cing, and so on.	"3RSSH, 3R" = "3RSSH"smooth the raw data, then get the residuals which will be smooth with "3R" filter. The results of the two procedures are added as the final smoothed values of the original data. "3RSSH, 3RSSH" = example of twicing, may be noted as "3RSSH, twice"

Source: Tukey (1977) and Quantitative Decisions (2004)

Some Criteria for Seasonal Adjustment

Desirable seasonal adjustment procedures possess practical criteria when used in adjusting time series data. Such characteristics are (1) minimum revisions, (2) smoothness, and (3) accuracy in estimation.

As time series data is updated and added on with the most recent data, it is much desired that previously published seasonally-adjusted results are not very different from the latest published adjusted data. This is a desirable property for statistical agencies, where consistency of their results is paramount for their credibility. This property is called the minimum revisions criterion. For comparison of minimum revision history among seasonal adjustment programs, the revision history mean absolute percentage error ($RHMAPE_K$) is used (Hungarian Central Statistics Office, 2007). A lower $RHMAPE_K$ for a specific program indicates that it is better in terms of minimal differences of latest published results from those that were previously published. The formula for $RHMAPE_K$ is shown below:

Revision History MAPE: $$RHMAPE_K = \frac{1}{T-K}\sum_{t=1}^{T-K}\left|\frac{SA_t^{(T-K)} - SA_t^{(T)}}{SA_t^{(T)}}\right| \qquad (6)$$

The term $SA_t^{(T-K)}$ is the SA value at time t when K recent periods were withheld in the adjustment, whilst $SA_t^{(T)}$ is the SA value when all periods were used. A smaller MAPE is desired so that there is minimum revision in seasonal adjustment of time series data.

Smoothness is also desired to be achieved in seasonally adjusted data, since this meant that estimated trend have reduced roughness, irregularities are relatively small on the average, and seasonal behavior have been isolated from the seasonally adjusted data. Smooth seasonally adjusted data is desirable for optimal seasonal adjustment. To judge smoothness between procedures, the statistic to be used is shown below (Bersales, 2010):

Smoothness Statistic: $$SM = \frac{1}{T}\sum_{t=2}^{T}\left|SA_t - SA_{t-1}\right| \qquad (7)$$

It is desired that SM is small so that smoothness is achieved from adjustment.

Accurate results are greatly desired in seasonal adjustment. Accurate estimation of the components makes analysis of seasonally adjusted data more credible. Study of accuracy is only possible in simulation experiments since non-seasonal components are unobserved in real time series data. As measure of accuracy among methods, the mean absolute percentage error (MAPE) for estimation of the non-seasonal is used. The formula used would be with respect to the prediction of the non-seasonal component as the extraction of it was the concern of estimation by the seasonal adjustment procedure. The formula for the MAPE was written below; let NS_t be the non-seasonal component (sum of the trend-cycle and irregular):

Estimation MAPE: $$MAPE = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{NS_t - SA_t}{NS_t}\right| \qquad (8)$$

The Monitoring and Quality Control Statistics of X-11-ARIMA

In addition to the features earlier discussed, X-11-based programs have quality control statistics that assess SA data with respect to the goodness of smoothing and extraction of components (Lothian and Morry, 1978). These statistics are called M statistics, which describes the compliance of the SA data with some assumptions and criteria for seasonal adjustment. The M statistics indicate on which criteria the seasonal adjustment has satisfied. The lower and upper bounds of the M statistics are 0 and 3 respectively. If any of the M statistics are less than 0 or more than 3, then the statistic is set to be 0 or 3 respectively. Failure is indicated when the M statistic is greater than 1. The adjustment results are rejected when all M statistics indicate failure. A summary to the M statistics is their weighted average, called the Q statistics, which indicates whether the SA data is acceptable on the collective criteria of seasonal adjustment that is assumed by X-11-based programs. Since no pre-adjustment procedures for the proposed seasonal adjustment method were used, Lothian and Morry (1978) provided the original formulas, whilst the research of Macadangdang (2006) posted some alternative methods for solving for M statistics in special cases.

Table 2. The M Statistics

Statistic Name	Criteria
M1: Relative Contribution of Irregular Component over Three Months	If M1≤1, then the irregular has minimal contribution to the changes over three months.
M2: Relative Contribution of Irregular to Variation of Stationary Component	If M2≤1, then the irregular behavior has minimal contribution to the variation of stationary behavior.
M3: Ratio of Mean Absolute Changes of the Irregular and the Trend-Cycle	If M3≤1, then the absolute changes due to irregularities are small enough relative to absolute changes in the trend cycle.
M4: Autocorrelation Based on the Standardized Average Duration of Runs	If M4≤1, then the irregular component has no autocorrelation.
M5: The Cyclical Dominance Statistic	If M5≤1, then the cycle component dominates over the irregular behavior.
M6: Ratio of Year-to-Year Changes in Irregular Compared to Seasonal Component	If M6≤1, then the annual irregular component changes are smaller relative to annual seasonal component changes.
M7: Statistic for Identifiable Seasonality	If M7≤1, then seasonality is identifiable (i.e., more stable than volatile)
M8: Fluctuation Size of the Seasonal Component throughout the Whole Series	If M8≤1, then there are very small differences between similar seasons in different years
M9: Average Linear Movement of the Seasonal Component throughout the Whole Series	If M9≤1, then the seasonal behavior has no absolute linear trend
M10: Size of Seasonal Component Fluctuations in Recent Years	Similar to M8, but for recent years
M11: Average Linear Movement of the Seasonal Component in Recent Years	Similar to M9, but for recent years

Source: Lothian and Morry (1978)

Composite statistics for overall acceptance and rejection are the Q statistics. To summarize the results of the M statistic, a Q statistic is the weighted average of the M statistic (Lothian and Morry, 1978):

Q Statistic:
$$Q = \frac{\sum_{i=1}^{11} w_i Mi}{\sum_{i=1}^{11} w_i} \tag{6}$$

The weights of the Q statistics corresponding to each M statistics are listed below in table (2), with modifications to the Q statistics dependent on the seasonal adjustment procedure and length of the data based on Lothian and Morry (1978). If the Q statistic is greater than one, then the seasonal adjustment results are rejected.

Table 3: Weights for Q Statistics

M Statistics	Q: Default Seasonal Component Estimation
M1	13
M2	13
M3	10
M4	5
M5	11
M6	10
M7	16
M8	7
M9	7
M10	4
M11	4

Source: Lothian and Morry (1978)

Proposed Seasonal Adjustment Procedure

The repeated median-based seasonal adjustment procedure is patterned from the X-11 family of moving average filters. Supposing that y_t is the original data with seasonal frequency*SEAS*:

Step 1 (a): Initial estimation of the trend component using a **"(SEAS+1) (SEAS-1), (SEAS+1) (SEAS-1), (SEAS+1) (SEAS-1), >, (SEAS+1) (SEAS-1)"** median filter. For example for quarterly data, a **"5 3, 5 3, 5 3, >, 5 3"** median filter was used. Let the initial trend be $y_t^{Trend(1)}$. The treatment of beginning and end values would be similar as discussed in Section 2.8: copying on for smoothing windows wider than 3 and the special smoothing method by Tukey (1977) for three-window median smoothing.

Step 1 (b): Initial estimation of the SI component via subtraction or division, depending on the structure for decomposition.

Step 1 (c): Initial estimation of seasonal component using a "**33 3**" seasonal median filter. A seasonal median filter of window size 3 is shown below:

3-term Seasonal Median: $$y_t^{Seas(1)} = median\left\{y_{t-SEAS}^{SI(1)}, y_t^{SI(1)}, y_{t+SEAS}^{SI(1)}\right\} \tag{9}$$

3-term Start Value Smoothing: $$y_1^{Seas(1)} = median\left\{3y_{SEAS+1}^{Seas(1)} - 2y_{2SEAS+1}^{Seas(1)}, y_1^{SI(1)}, y_{SEAS+1}^{Seas(1)}\right\} \tag{10}$$

3-term End Value Smoothing: $$y_T^{Seas(1)} = median\left\{y_{T-SEAS}^{Seas(1)}, y_T^{SI(1)}, 3y_{T-SEAS}^{Seas(1)} - 2y_{T-2SEAS}^{Seas(1)}\right\} \tag{11}$$

The seasonal median introduced in this paper has similar additional procedures as the basic median filter by Tukey (1977), with the alteration of time difference between observations with length*SEAS* compared time difference of 1 for the basic median filter.

Step 1 (d): Initial estimation of SA data with the seasonal components not adjusted since they are smoothed with respect to trend.

Step 2 (a): Optimal trend estimation using the **"(SEAS+1)R (SEAS-1)RSSH, (SEAS+1)R (SEAS-1)RSSH, (SEAS+1)R (SEAS-1)RSSH"** median filter on the initial SA data. The reroughed "**3RSSH"** median smooth (twiced or thriced) was the optimal smoother of Tukey

(1977) for annual data examples. To adapt for seasonal periods of the data, a season-dependent width was introduced.

Step 2 (b): Estimation of the SI component from the original series using the optimal trend estimates.

Step 2 (c): Estimation of the seasonal component from SI components of Step 2 (b) using a seasonal **"5 5 5 3 3 3"** median filter. For the 5-term median smoothing, beginning or end values were copied on to the resulting smooth series.

Step 2 (d): Estimation of the final SA data using the seasonal component of Step 2 (c).

Step 3: Estimation of the final trend-cycle estimate using the **"(SEAS+1)R (SEAS-1)RSSH, (SEAS+1)R (SEAS-1)RSSH, (SEAS+1)R (SEAS-1)RSSH"** median filter. The irregular component for the procedure is equal to the difference or quotient of the final SA component and the final trend-cycle estimate

Comparison with X-11 Method

The proposed methodology of the paper closely resembles X-11. These differences in methodology are shown below.To substitute for the Henderson MA used for X-11-ARIMA, the optimal smoother would be the "**3RSSH**" smoother of Tukey (1977). To make "**3RSSH**" optimal, it was thriced and the window width was made dependent of the number of seasons.

Table 4. Comparisons between the Proposed Seasonal Adjustment and X-11-ARIMA in Default Setting

Stage of Adjustment	Proposed Adjustment	X-11-ARIMA Seasonal Adjustment (Default Settings)**
Initial Trend Estimate	Initial Smooth: **"(SEAS+1) (SEAS-1), (SEAS+1) (SEAS-1), (SEAS+1) (SEAS-1), >, (SEAS+1) (SEAS-1)"** median filter	Initial Trend Estimate: A (SEAS+1)-term centered MA, with the middle (SEAS-1) terms having double weights compared to the ends.
Seasonal Component Estimation	Initial Seasonal Smooth: "**3 3 3**" seasonal median filter Secondary Seasonal Smooth: "**5 5 5 3 3 3**" seasonal median filter	Initial Seasonal Estimate: 3x3 seasonal MA Secondary Seasonal Estimate: 5x3 seasonal MA
Trend Smooth	Trend Estimation: **"(SEAS+1)R (SEAS-1)RSSH, (SEAS+1)R (SEAS-1)RSSH, (SEAS+1)R (SEAS-1)RSSH"** median filter	Trend Estimate: (SEAS+1) Henderson MA

**Source: Ghysels and Osborn (2001)

Discussion of Simulation Study

To facilitate comparisons between the proposed procedure and default X-11ARIMA seasonal adjustment, simulation experiments are conducted and RHMAPE, SM, and estimation MAPE statistics are gathered and analyzed from them.The table below shows the different fixed and variable cases for the simulation experiment. The study considers the cases for its scope yet future research may seek to expand the cases for further investigation.

Table 5. Conditions for Simulation Experiments

Scenarios	Cases	Specifications
Fixed Conditions	Number of Series per Case	100 series, each with 120 periods (30 years for quarterly, 10 years for monthly)
	Trend	Exponential Trend: $T_t = 40(1.03)^t$
	Cycle-Irregular	AR(1) Model: $CI_t = 0.8CI_{t-1} + \varepsilon_t$, $CI_0 = 0$ and white noise process $\varepsilon_t \sim WN(0,1)$
Variable Conditions	Structure of Seasonal Series	1. Additive: $y_t = NS_t + S_t$ 2. Multiplicative: $y_t = NS_t \times S_t$ Where NS_t =non-seasonal component, S_t =seasonal component
	Seasonal Frequency	1. Quarterly (S=4) 2. Monthly (S=12)

Table 5.Conditions for Simulation Experiments (cont.)

Scenarios	Cases	Specifications
Variable Conditions	Seasonal Volatility (when there is no seasonality, there is no seasonal volatility)	1. No Seasonal Volatility 2. With Seasonal Volatility: $SGARCH(1,1,S)$ model $\varepsilon_{i,t} = v_{i,t}\sqrt{h_{i,t}}$ $v_{i,t} \sim WN(0,1)$ $h_{i,t} = 0.01 + 0.45\varepsilon_{i,t-S}^2 + 0.5h_{i,t-S}$ $\varepsilon_{i,0} = 0, h_{i,0} = 0.01$ For additive cases: $S_t^{Vol} = S_t + \frac{1}{3}\left(\sqrt{SGARCH} - Average\left(\sqrt{SGARCH}\right)\right)$ $(Weak)$ $S_t^{Vol} = S_t + \left(\sqrt{SGARCH} - Average\left(\sqrt{SGARCH}\right)\right)$ $(Strong)$ For multiplicative cases: $S_t^{Vol} = S_t\left[1 + \left(\frac{\sqrt{SGARCH} - Average\left(\sqrt{SGARCH}\right)}{50}\right)\right]$ $(Weak)$ $S_t^{Vol} = S_t\left[1 + \left(\frac{\sqrt{SGARCH} - Average\left(\sqrt{SGARCH}\right)}{20}\right)\right]$ $(Strong)$
	Magnitude of Seasonality	1. No Seasonality: Additive $S_t = 0$; Multiplicative $S_t = 1$ 2. Weak Seasonality 3. Strong Seasonality

Quarterly Seasonal Effects

Additive		Multiplicative	
Weak	Strong	Weak	Strong
-5	-25	99.0%	87.5%
-2	-10	99.6%	95.0%
-3	-15	99.4%	92.5%
10	50	102.0%	125.0%

Monthly Seasonal Effects

		2	10	100.4%	110.0%
		-3	-15	99.4%	85.0%
		-6	-30	98.8%	70.0%
		2	10	100.4%	110.0%
		3	15	100.6%	115.0%
		2	10	100.4%	110.0%
		3	15	100.6%	115.0%
		-4	-20	99.2%	80.0%
		-1	-5	99.8%	95.0%
		0	0	100.0%	100.0%
		-4	-20	99.2%	80.0%
		6	30	101.2%	130.0%

Real World Data Application

After performing simulation experiments, the method is demonstrated to real Philippine seasonal economic time series data. The following datasets with their details and seasonality features in terms of strength of seasonal volatility are expressed in the table below.

Table 6. Philippine Time Series Data for Evaluation of Seasonal Adjustment Methods

Period	Time Series Data	Source Agency	Data Span	Unit	Features
Quarterly	Total Imports (Impts)	NSCB	1981 First Quarter - 2010 Second Quarter	in Million Pesos, 1985 Constant Prices	Weak Volatile Seasonality
	Gross Value-Added of the Agricultural and Forestry Sector	NSCB	1981 First Quarter - 2010 Second Quarter	in Million Pesos, 1985 Constant Prices	Strong Volatile Seasonality
Monthly	Consumer Price Index on Food, Beverage, and Tobacco (CPI-FBT)	BangkoSentralngPilipinas (BSP)	January 1994 – February 2010	None, Base Year 2000=100	Weak Seasonal Volatility
	Overseas Filipinos' Remittances (OFRemit)	BSP	January 1989 – August 2010	in US$ Thousands	Strong Seasonal Volatility

The time series would be subjected to both default X-11 and the median-based seasonal adjustment methods. Each series would be subjected to both additive and multiplicative seasonal structures. The M and Q statistics of Lothian and Morry (1978), revision history MAPE, and SM statistic would be derived for both procedures for comparison.

The Default X-11 Settings

The default setting made for all seasonal adjustment procedures under X-11-ARIMA are as follows: (1) no trading day adjustments, (2) X-11 seasonal adjustment default, (3) 13-term Henderson (for monthly) or 5-term Henderson (quarterly), (4) no data transformations, (5) no ARIMA specification for model building, but default ARIMA model for backcasts and forecasts unchanged, and (6) no regressors. The defaults would facilitate reasonable comparisons such that the procedures are of equal footing.

Simulation Results

The table below shows the results of the simulation experiment in evaluation and comparison of the median-based seasonal adjustment (MX11) procedure and default Census X-11 (CX11) results. The difference (Diff) value listed below is the difference of the averages of statistics between MX11 and CX11, while the percentage (%) seen below is the percentage of series of which MX11 had a better results compared to CX11.

Table 7. Evaluation Statistics with Respect to Seasonal Behavior and Frequency

Seasonality Features			Additive Seasonal Adjustment						Multiplicative Seasonal Adjustment					
			RH_MAPE		SM		MAPE		RH_MAPE		SM		MAPE	
Magnitude	Freq	Structure	Diff	%	Diff	%	Diff	%	Diff	%	Diff	%	Diff	%
No Seasonal Pattern	Quarterly	Additive	-1.136%	29%	0.010	46%	-0.146%	92%	-1.134%	23%	0.004	50%	0.086%	17%
		Multiplicative	-1.604%	31%	0.059	51%	0.044%	48%	-1.588%	33%	0.135	46%	0.117%	27%
	Monthly	Additive	-1.308%	86%	-0.045	69%	-0.344%	94%	-1.240%	52%	-0.069	74%	-0.128%	70%
		Multiplicative	-1.947%	94%	-0.699	75%	-0.279%	77%	-1.566%	58%	-0.329	60%	-0.482%	98%
Weak Stable Seasonal Pattern	Quarterly	Additive	-0.821%	24%	-0.214	42%	-0.275%	100%	-0.819%	24%	-0.215	53%	0.132%	21%
		Multiplicative	-1.094%	31%	-0.082	46%	-0.002%	58%	-1.077%	30%	-0.022	40%	-0.019%	58%
	Monthly	Additive	-0.978%	81%	-0.254	51%	-0.472%	98%	-0.922%	44%	-0.284	63%	-0.085%	78%
		Multiplicative	-1.382%	94%	-0.413	81%	-0.245%	74%	-1.109%	58%	-0.221	64%	-0.486%	96%
Strong Stable Seasonal Pattern	Quarterly	Additive	-1.121%	17%	-0.523	12%	0.075%	38%	-1.122%	24%	-0.185	68%	0.320%	0%
		Multiplicative	-1.600%	32%	-0.500	63%	0.921%	0%	-1.584%	19%	-0.045	42%	-0.018%	55%
	Monthly	Additive	-1.437%	43%	-0.222	31%	-0.101%	76%	-1.715%	81%	-0.036	98%	-0.140%	100%
		Multiplicative	-2.046%	100%	-0.947	100%	-0.327%	100%	-1.752%	25%	-0.286	53%	-0.121%	74%
Weak Seasonal Volatility	Quarterly	Additive	-0.812%	16%	-0.617	36%	-0.275%	100%	-0.809%	12%	-0.247	48%	0.137%	20%
		Multiplicative	-1.084%	32%	-0.584	37%	-0.022%	53%	-1.068%	19%	-0.129	45%	-0.025%	58%
	Monthly	Additive	-1.145%	79%	-0.272	43%	-0.638%	100%	-1.404%	47%	-0.033	71%	-0.086%	74%
		Multiplicative	-1.481%	96%	-0.563	83%	-0.320%	86%	-1.329%	71%	-0.070	56%	-0.573%	100%
Strong Seasonal Volatility	Quarterly	Additive	-1.065%	15%	0.188	8%	0.117%	26%	-1.066%	33%	0.335	73%	0.325%	0%
		Multiplicative	-1.525%	18%	-0.127	80%	0.882%	0%	-1.512%	8%	0.110	25%	-0.010%	58%
	Monthly	Additive	-1.292%	53%	-0.008	40%	-0.248%	90%	-1.180%	90%	0.317	100%	-0.136%	100%
		Multiplicative	-1.906%	100%	-1.049	100%	-0.330%	100%	-1.441%	34%	-0.266	39%	-0.279%	93%

A second table below shows the marginal results, looking at the overall average differences and relative frequencies over every individual scenario per case in the simulation experiment, ignoring all other cases by averaging.

Table 8. Evaluation Statistics with Respect to Seasonal Behavior and Frequency

	Additive						Multiplicative					
	RH_MAPE		SM		MAPE		RH_MAPE		SM		MAPE	
	Diff	%	Diff	%	Diff	%	Diff	%	Diff	%	Diff	%
No Seasonal Behavior	-1.499%	60%	-0.169	60%	-0.181%	78%	-1.382%	42%	-0.065	58%	-0.102%	53%
Stable Seasonal Pattern	-1.310%	53%	-0.394	53%	-0.053%	68%	-1.263%	38%	-0.162	60%	-0.052%	60%
Seasonal Volatility	-1.289%	51%	-0.379	53%	-0.104%	69%	-1.226%	39%	0.002	57%	-0.081%	63%
Weak Seasonal Pattern	-1.100%	57%	-0.375	52%	-0.281%	84%	-1.067%	38%	-0.153	55%	-0.126%	63%
Strong Seasonal Pattern	-1.499%	47%	-0.398	54%	0.124%	54%	-1.422%	39%	-0.007	62%	-0.007%	60%
Quarterly	-1.186%	25%	-0.239	42%	0.132%	52%	-1.178%	23%	-0.026	49%	0.105%	31%
Monthly	-1.357%	75%	-0.407	61%	-0.300%	81%	-1.242%	51%	-0.116	62%	-0.229%	80%
Additive Structure	-1.011%	40%	-0.178	34%	-0.210%	74%	-1.037%	39%	-0.038	63%	0.039%	44%
Multiplicative Structure	-1.424%	57%	-0.446	65%	0.029%	54%	-1.275%	32%	-0.102	43%	-0.172%	65%

In both additive and multiplicative methods for seasonal adjustment, MX11 was better in terms of revisions than CX11 in terms of average difference of RH MAPE. In terms of relative frequency, MX11 was better in revisions than CX11 in the additive seasonal adjustment, but the reverse is observed in multiplicative adjustment, where CX11 was better in revision history. In terms of seasonal frequency, CX11 was better in revision history for quarterly data, while for monthly data X11 was better, both in terms of average difference and relative frequency.

On the average, MX11 methods were comparatively smoother than CX11. In terms of relative frequency, CX11 was better in quarterly data but for monthly data MX11 had better statistics. In additive seasonal adjustment, MX11 was generally better than CX11 on the average, yet in multiplicative seasonal adjustment, seasonal volatility may make CX11 better, but not in frequency.

In accuracy of the estimates of the non-seasonal component, MX11 was frequently more accurate than CX11 based on percentages when an additive seasonal adjustment method was used. CX11 would tend to be better more frequently in quarterly cases for multiplicative seasonal adjustment.

In summary, additive MX11 seasonal was better in revision history, smoothness, and accuracy compared to additive CX11 adjustment. Generally, MX11 was better in monthly data compared to quarterly data. In cases of seasonal volatility, MX11 was frequently better than CX11 in additive and multiplicative seasonal adjustment.

Real Data Results

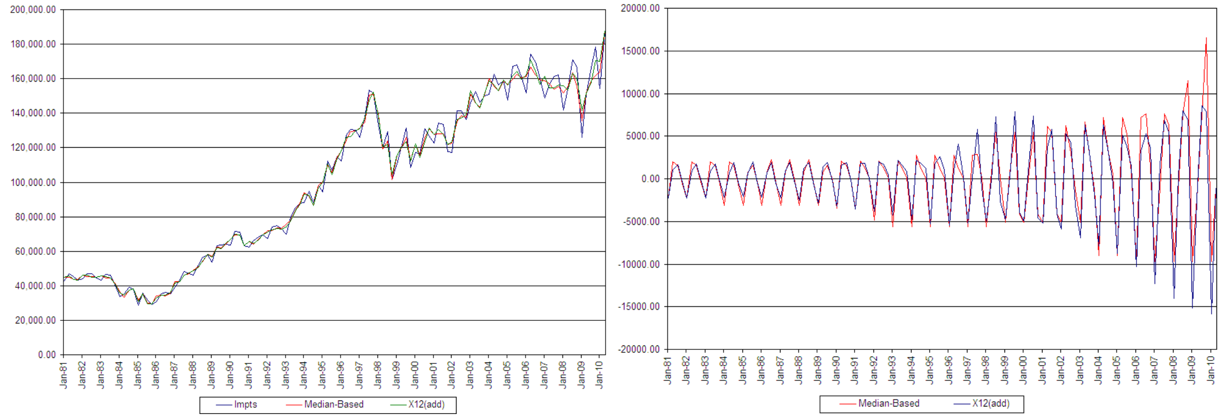
The adjustment procedures are then compared in real data applications. Philippine economic time series data is used in this exercise and comparisons of results using graphs and statistics tables are made.

Total Imports

Total imports data have been indicated by Bersales (2010) to have nonlinear seasonal structure in the form of seasonal volatility, in which seasonal behavior between years were not of similar magnitude. Comparisons of results between median filtering and X-11-ARIMA results were conducted. However, deviating from Bersales (2010), quarterly figures of imports generated by the National Statistical Coordination Board from the Philippine System of National Accounts was used for evaluation and comparison.

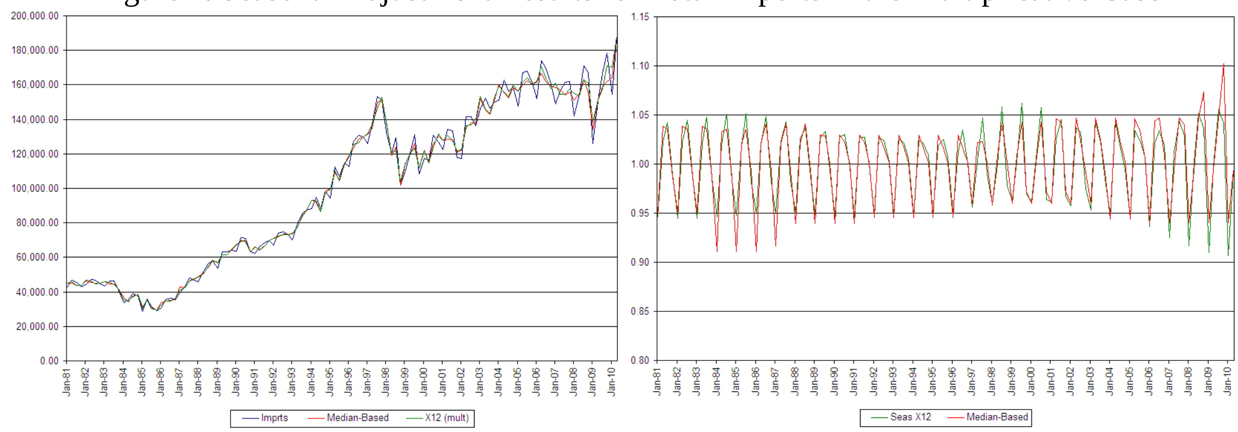
Looking at the graphs of figure (1), the original imports series has some seasonal pattern that have slightly changed, such of which was that for periods earlier than January 1997, seasonal pattern was not strong, yet post-1997 data would later feature stronger seasonal behavior. Seasonal component estimates reflect this analysis of the seasonal behavior in which case the strength did become higher in post-1997.

Figure 1. Seasonal Adjustment Results for Total Imports in the Additive Case



Original and Seasonally Adjusted Series Estimates of the Seasonal Component

Figure 2. Seasonal Adjustment Results for Total Imports in the Multiplicative Case



Original and Seasonally Adjusted Series Estimates of the Seasonal Component

Table 9. Evaluation Statistics for Seasonal Adjustment of Total Imports Data

Total Imports Seasonal Adjustment				
Evaluation Statistics	Median-Based		Default X-11-ARIMA	
	Additive	Multiplicative	Additive	Multiplicative
M1	3.000	0.000	1.336	1.057
M2	0.815	0.000	0.210	0.074
M3	3.000	0.000	0.456	0.301
M4	0.480	0.649	0.740	0.398
M5	1.914	0.200	0.421	0.429
M6	1.774	2.980	0.586	0.266
M7	2.478	1.454	0.556	0.373
M8	1.300	1.137	1.369	1.236
M9	0.637	0.128	0.591	0.309
M10	3.000	2.413	3.000	2.581
M11	3.000	2.413	3.000	2.581
Q	1.980	0.867	0.855	0.645
RH MAPE	0.17176%	0.15390%	0.20305%	0.17117%
SM	4585.2867	4658.5931	4654.6001	4748.2149

Based on the results for total imports in table (7), generally multiplicative models would generate better quality of adjustment. In comparison between MMX11 and MCX11 seasonal adjustment, the Q statistic would indicate the latter to have better results. The RH MAPE and SM statistics indicated favor with the MMX11 as better than MCX11 in the seasonal adjustment of imports series data.

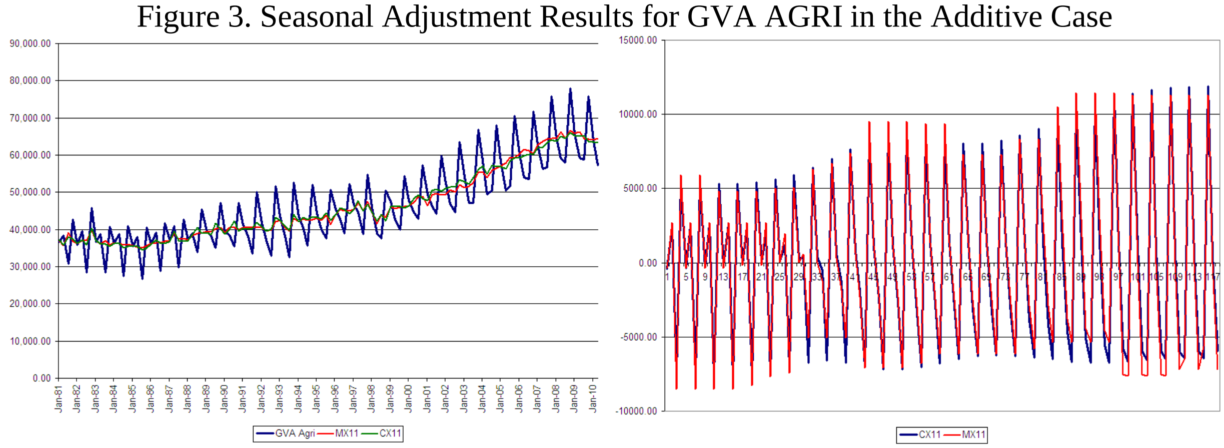
Gross Value Added (GVA) in the Agriculture and Forestry Sector

The study of gross value-added in the agricultural sector is important since production in this sector that provides raw materials to other sector of economic development. Seasonal analysis of agricultural activity is of interest since the industry is most susceptible to seasonal climate. Figures (3) and (4) presented the graphical results of adjustment for additive and multiplicative decomposition respectively, whilst table (8) presents the evaluation statistics for GVA AGRI.

Observing the behavior of the original series, an increasing trend of value-added for agriculture was observed. Seasonal behavior seemed to change in the agricultural sector, with peaks frequently slightly changing in magnitude. Seasonal volatility may be observed from the GVA.

Results of seasonally-adjusted data and seasonal estimates observed slight similarity in the results of MX11 and CX11. Looking at the evaluation statistics, generally all estimates from both procedures and decompositions were acceptable. However, it was observed that MMX11 provided lower Q values and was smoother than CX11 adjustment. All revision history statistics were acceptable since all were less than 1.00% but CX11 had better revision history.

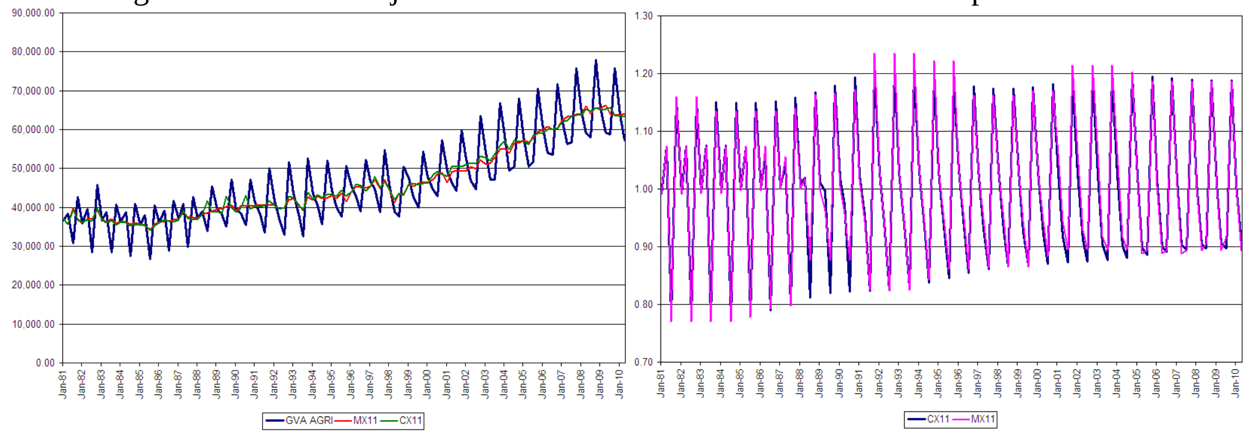
Figure 3. Seasonal Adjustment Results for GVA AGRI in the Additive Case



Original and Seasonally Adjusted Series

Estimates of the Seasonal Component

Figure 4. Seasonal Adjustment Results for GVA AGRI in the Multiplicative Case



Original and Seasonally Adjusted Series Estimates of the Seasonal Component

Table 10. Evaluation Statistics for Seasonal Adjustment of GVA AGRI Data

GVA of Agriculture and Forestry Seasonal Adjustment				
Evaluation Statistics	Median-Based		Default X-11-ARIMA	
	Additive	Multiplicative	Additive	Multiplicative
M1	0.189	0.001	0.055	0.056
M2	0.216	0.000	0.049	0.070
M3	3.000	0.000	0.792	0.967
M4	0.311	0.311	1.082	0.911
M5	2.199	0.200	0.856	0.956
M6	0.461	0.524	0.695	0.598
M7	0.204	0.174	0.167	0.133
M8	0.602	0.598	0.349	0.344
M9	0.263	0.082	0.252	0.231
M10	0.556	0.215	0.248	0.228
M11	0.556	0.215	0.248	0.228
Q	0.794	0.183	0.399	0.403
RH MAPE	0.19354%	0.10664%	0.0230%	0.0443%
SM	1001.2422	993.2030	1169.1524	1208.0445

Consumer Price Index on Food, Beverage, and Tobacco

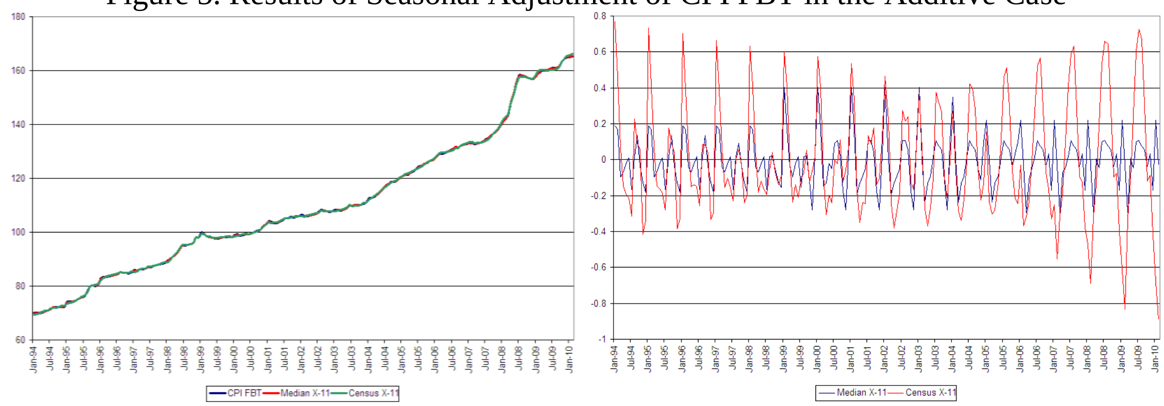
The price index on food has been indicated for seasonal adjustment by Dimaandal and Asence (1993), as food prices tend to be affected by climate patterns, especially with crop foods such as grains, fruits, and vegetables. It has been indicated to have weak seasonality, yet Redoblado (2005) had concluded that there is some instability in the seasonal pattern of the price index. Weak seasonal volatility may be evident from the CPI FBT data. Table (9) displays the evaluation statistics for using MX11 and CX11 in different decomposition structures. Figures (5) and (6) display graphical results for additive and multiplicative decompositions respectively.

Differences in the seasonal estimates were seen between the MX11 and CX11 procedures. In both additive and multiplicative decompositions, CX11 estimates were unstable, such that a slow seasonal shift was emerging from the series, were the peaks in January slow change to low levels and a reverse occurred in July. Changing amplitude of seasonal estimates was also observed in CX11 procedures, but this became weaker in multiplicative decomposition. MX11 seasonal estimates in both decomposition structures were more stable, with a seemingly constant band of amplitude. There are some regimes of seasonal shifts evident in the MX11 estimates. Three shifts may be observed from these seasonal estimates: (i) 1994-1998, (ii) 1998-2004, and (iii) 2004 to

2010. In 1994-1998, January and February prices were highest, and prices in mid-year were relatively weaker. In 1998-2004, seasonal patterns changed with relatively larger amplitude; mid-year prices were then of higher price compared to the previous regime. After 2004, the amplitude has reduced back, yet the pattern of mid-year prices seemed to be similar to the previous era.

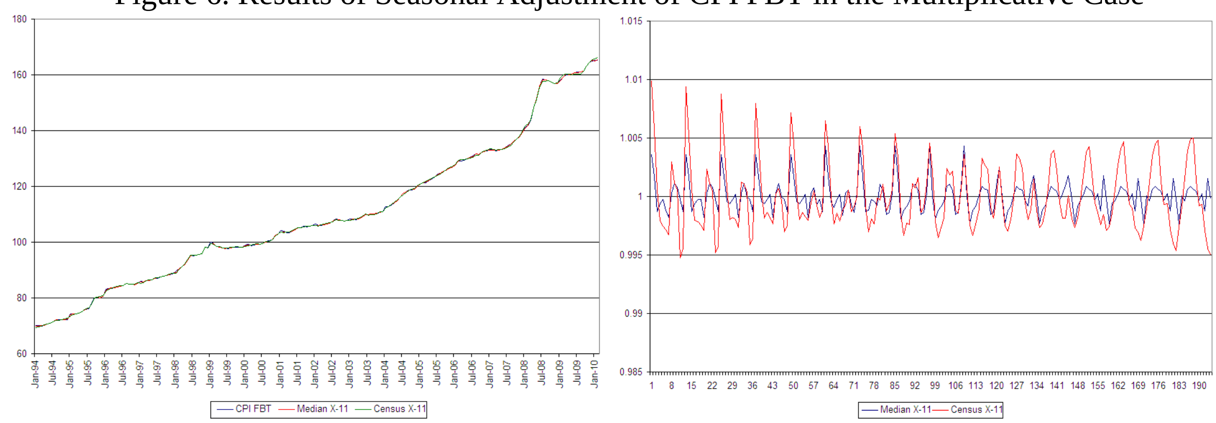
Basing from the features of the CPI data, a weak but volatile seasonality with trend shift in the ends may be observed. Stable constant variation may be observed from the series. Structure was relatively unclear, yet seasonal factors stabilized in cases of multiplicative adjustment, thus an assumption of multiplicative structure would be made. Based on the RH MAPE and SM statistics, MMX11 estimates were better than CX11 methods. From the seasonal adjustment, favorable results were observed for the median-based methodology.

Figure 5. Results of Seasonal Adjustment of CPI FBT in the Additive Case



Seasonally Adjusted and Original Series | Seasonal Component Estimates

Figure 6. Results of Seasonal Adjustment of CPI FBT in the Multiplicative Case



Seasonally Adjusted and Original Series | Seasonal Component Estimates

Table 11. Evaluation Statistics for the Seasonal Adjustment of CPI FBT

CPI for Food, Beverage and Tobacco Seasonal Adjustment				
Evaluation Statistics	Median-Based		Default X-11-ARIMA	
	Additive	Multiplicative	Additive	Multiplicative
M1	0.674	0.498	0.230	0.233
M2	0.045	0.063	0.016	0.030
M3	0.000	0.000	0.000	0.000
M4	1.476	1.343	1.261	1.261
M5	0.000	0.000	0.000	0.000
M6	3.000	3.000	0.037	0.063
M7	2.960	2.279	2.380	1.865
M8	1.268	1.183	1.717	1.688
M9	0.468	0.528	1.143	1.077
M10	1.227	1.050	2.452	1.882
M11	1.227	1.050	2.437	1.867
Q	1.160	1.009	0.875	0.745
RH MAPE	0.0121%	0.0112%	0.0292%	0.0226%
SM	0.5611	0.5617	0.5628	0.5635

Overseas Filipinos' Remittances

The results of seasonal adjustment of overseas Filipinos' remittances data were shown below, with graph for additive and multiplicative cases, respectively in figures (7) and (8), whilst table (10) show the results of evaluation statistics. Hypotheses on the seasonal behavior of remittances to the country such as in December, May or June may be explored through seasonal adjustment.

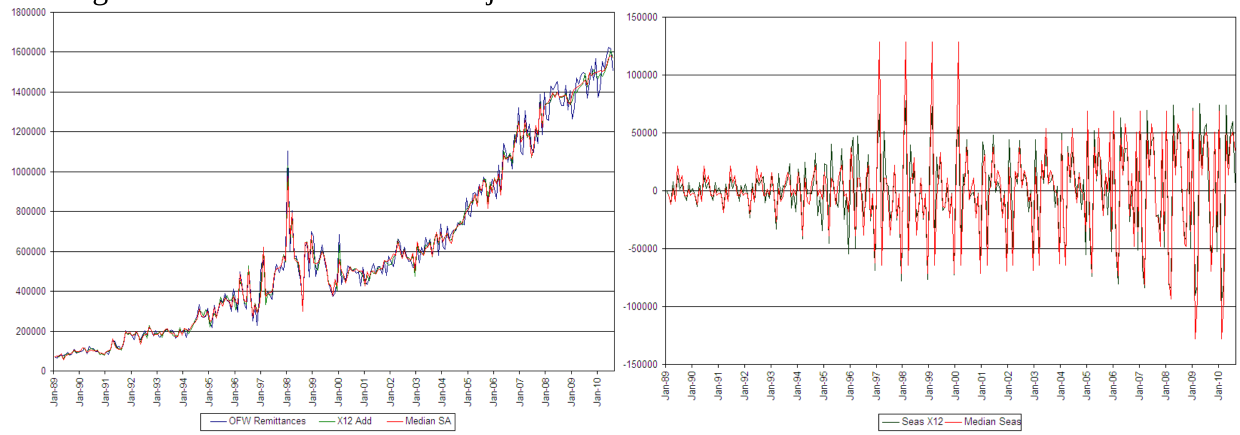
Notable features of overseas remittances data were its nonconstant volatility throughout of the series, shown in graphs below. Seasonal patterns were very unclear to be seen in time series, possibly masked by the sudden changes in overseas remittances behavior, but it would be explored later.

From the graphs of the seasonal components, some differences may be noticed between the two procedures. Seasonal values before 1997 between the two methods were very different from one another, but the difference became miniscule through time. High estimates of seasonal behavior were observed by the median adjustment from 1997 to 2000, related to the volatility of the Asian Financial Crisis, yet this tempers at the beginning of 2000. Seasonal behavior seemed to changes in four periods: (1) before 1993, (2) 1993-1997, (3) 1997-2000, and (4) 2000 and after. Breaks in seasonal adjustment were apparent in the overseas remittances data. Currently, seasonal estimates converge to their current pattern of relative highs in March to June, July, October, and December, and relative lows on January, February, August, September, and November.

OFW remittances were observed to have volatile seasonality and nonconstant variations in time. An innovational outlier in the middle of the series may be posited. Weak seasonality would be assumed based on the range of the seasonal behavior in recent years. More stable seasonal results were observed in the multiplicative decomposition from the seasonal estimates in figure (10), thus an assumption of multiplicative structure was made. Based on the results of the evaluation statistics, from the Q statistic, seasonal adjustment using MMX11 seemed to give the optimal result. It was the only result that could be conditionally accepted when observing the two M statistics to which it has failed. The results of seasonal adjustment using CX11 were rejected for the series. Median-based methods were able to gather identifiable seasonality as indicated by the

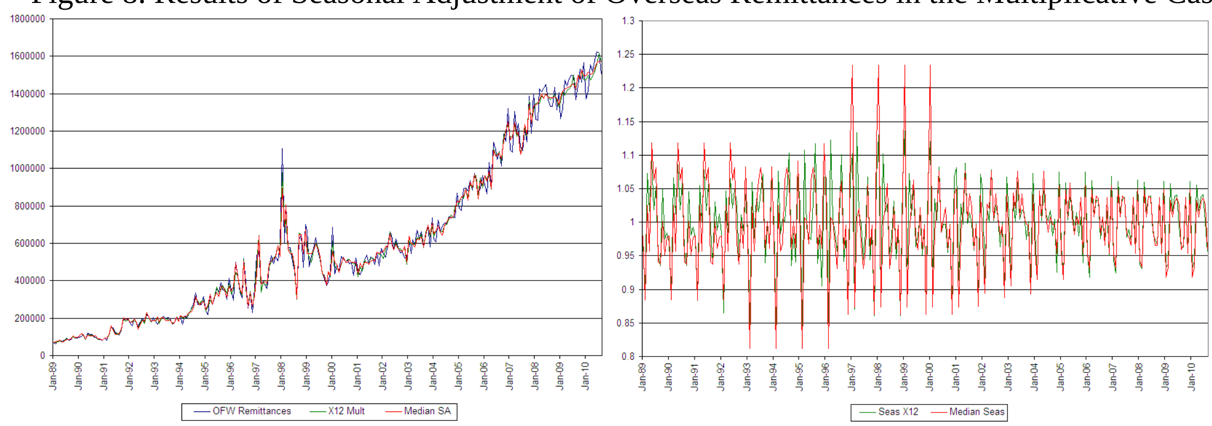
M7 statistic, whilst CX11 methods failed the criterion. Median-based methods were smoother. Revision histories were small for both procedures, yet CX11 had better revision.

Figure 7. Results of Seasonal Adjustment of Overseas Remittances in the Additive Case



Seasonally Adjusted and Original Series   Seasonal Component Estimates

Figure 8. Results of Seasonal Adjustment of Overseas Remittances in the Multiplicative Case



Seasonally Adjusted and Original Series   Seasonal Component Estimates

Table 12. Evaluation Statistics for the Seasonal Adjustment of Overseas Remittances

Overseas Remittances Seasonal Adjustment				
Evaluation Statistics	Median-Based		Default X-11-ARIMA	
	Additive	Multiplicative	Additive	Multiplicative
M1	3.000	0.000	2.357	2.495
M2	0.157	0.000	0.310	0.635
M3	2.687	0.000	0.753	0.685
M4	0.761	0.647	0.057	0.344
M5	1.378	0.000	0.779	0.746
M6	1.691	2.681	0.141	0.309
M7	0.204	0.231	1.552	1.461
M8	1.509	1.569	1.538	1.716
M9	0.537	0.407	0.683	0.258
M10	0.859	0.380	2.058	1.309
M11	2.244	0.861	1.942	0.869
Q	1.338	0.525	1.088	1.065
RH MAPE	0.171810%	0.303758%	0.17262%	0.13490%
SM	39982.85226	39462.7942	43041.3852	42021.237

Conclusions

From simulation experiments, MX11 methods have favorable properties of minimum revision changes, smoothness, and accuracy in estimation of the non-seasonal components in the presence of seasonal volatility in time series data. Future improvements in the MX11 procedure should be made especially in the quarterly time series data where CX11 tends to be better in seasonal adjustment.

From the assessment through simulation experiments, the procedure is applied to real Philippine data and compared with results of CX11, where the results show that the MX11 is a viable option in seasonal adjustment in the presence of unstable seasonal behavior.

The research opens the field for seasonal adjustment using robust nonlinear filtering methods in the face of non-linear and unstable conditions in seasonal time series data. Myriads of procedures can be created and mixing of procedures provide infinite possibilities in the search for the appropriate seasonal adjustment procedure.

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